**Chapter 14**

**Algebraic Thinking: Generalizations, Patterns, and Functions**

We are going to discuss the different methods that we read about in the text and how they align with current instruction or instruction that we may have experienced during our elementary or middle school years. Fill free to, at any time, make a comment or voice your opinion on a certain topic that we are discussing. We have all read the same chapter, but I am sure that we each picked up something different or had a better understanding of a certain topic, but with discussion we could learn from each other.

The algebra envisioned for these grades-and for high-school as well-is not the algebra that you most likely experienced in high-school (Van De Walle, Karp, Bay-Williams, pg. 254).

One of the best tools they can have is a deep understanding of the number system, its operations, and the properties related to those operations (Seeley & Schielack, *Curriculum Focal Points,* pg. 266).

**Generalization from Arithmetic and from Patterns**

Generalization with Addition

* Figure 14.1 monkeys and trees problem
  + Decompose and recompose numbers-generalizing that increasing the number in the small tree by one means reducing the number in the large tree by one.
    - Students may be asked to find all the ways the monkey can be in the two trees. Many will forget about using 0, other children will use each number from 0 to 7.
    - Inclusion of symbols is important for older children. Mice in the blue or green cage problem. b+g=8, 8-g=b, 8-b=g.
* Planning in advance- thinking of what questions you could ask to help the students think about the problem and what it is asking.

Generalization with the Hundreds Chart

* The hundred chart is a rich tool for exploring number relationships and should not only be used as a devise for teaching numeration.

Generalization Through Exploring a Pattern

* Find generalization in a growing pattern.
  + Examine only one growth step of a physical pattern and ask students to find a method of counting the elements without just counting by ones. Example: Activity 14.1- The Border Problem. (10x10 pool: 4x9, 4x8+4, 4x10-4, 100-64)
* Important idea in generalization is recognizing a new situation where it can apply and adapting it appropriately.

**Meaningful Use of Symbols**

When students are unsuccessful in algebra, it may be because they do not have a deep understanding of the symbols that they are using. Symbols should be seen as useful tools for solving important, real problems.

The Meaning of the Equal Sign

* Students should come to know the = symbol to indicate “equivalence” instead of “the answer is”.
* It is important for students to see, understand, and symbolize the relationship in our number system. The = sign shows these relationships. For example: 6 x 7=5 x 7 + 7, this is not only the fact strategy but also shows distributive property.
* If students have trouble understanding the = sign, they usually have difficulties with they are encountered with algebraic expression.
* Develop the concept of the equal sign concretely: use shapes and a balance. Activity 14.2 Seesaw Students and Activity 14.3 What Do You Know About Shapes? Pg. 258. After the students explore shapes then they can move on to numbers. The balance shows the students that if you add or subtract from one side, you have to do the same for the other side.
* Explore equations as true/false or open sentences. Inequalities and representations should be discussed.
* Relational thinking-when a student observes and uses relationships between two sides of the equal sign instead of computation. Select equations that elicit good thinking and challenges rather than computation. Use large numbers that make computation difficult, in order to push them towards rational thinking.
* Ask students to write their own open sentences to solidify their understanding.

The Meaning of Variables

Variables can be used as unique unknown values or as quantities that vary.

Variables used as unknown values:

* Usually the box is used for a missing variable in open sentences. Early on, use letters and ask students what number the letter could stand for to make the sentence true, this involves relational thinking.
* Activity 14.8- Balls, Balls, Balls, is a concrete activity because each variable represented an unknown value. You can also use a balance to serve as a concrete value. This helps build a foundation for working with equations later.

Simplifying expressions and equations:

Knowing how to simplify and recognize equivalent fractions are essential skills to working algebra.

* Have students look at simplifications that have errors and explain how to fix these error.

Variables used as quantities that vary:

The concept that variables can represent more than one missing value is not well understood or present in elementary or middle schools.

* Use context to help students make the connections better. Ask students to tie each number back to the context.
* It is also important to include decimal and fraction values in the exploration of variables. Use concrete and visual experiences to mix these.

**Making Structure in the Number System Explicit**

* Have students examine properties and structures explicitly and express them in general terms without reference to specific numbers. For example: 394 +176 = N +394, the student would say that N must be 176 because it is the same.

Making Conjectures about Properties

* Develop the aspects of properties in a conceptual manner, focusing on examples to guide students to generalize. This way you are not just presenting them and asking them to use them but introducing them first.
* Instead of responding with answers, ask questions. Do you think this will always be true? How can we find out?
* Push students to reason with logic and not be content with appeals to authority or example.
* Have students write their ideas before sharing with the class; you will have more to participate with discussion.

**Study of Patterns and Functions**

Repeating Patterns

* Draw simple shape pattern on the board and extend them in class discussion.
* It is Important that students see the importance of patterns in the world around them. (Sunday School-necklace)

Growing Patterns

* Should begin simple but then become more complicated.
* It is important to include fractions and decimals when working with growing patterns.
* Graphs provide visuals that allow students to see relationships among growing patterns. Consider strings of a single color of pattern blocks and the corresponding perimeters.
* Graphs offer opportunities to compare growing patterns. Ask students to discuss how to get from one coordinate to the next or what information can be found on a graph. Make sure students can link the graph to the context.
* It is fun to let the students draw there graphs on transparencies without identifying the situation and let the others see if they can determine it.

**Mathematical Modeling**

Mathematical modeling should not be confused with models that use manipulatives or visuals for building a pattern.

**Teaching Considerations**

Emphasize Appropriate Algebra Vocabulary

A large part of understanding mathematics is the ability to communicate mathematically, so it is important to use appropriate terminology in teaching algebra.

* Create word walls or have the students keep a journal, especially English Language Learners.

Multiple Representations

Provide different ways of looking or thinking about a function.

* Context, table, verbal description, symbols, and graphs.
* Make the movement among representations, not a rote procedure but making sense of a function.