Aly Grimes October 10, 2011

Chapter 18 Pedagogy: Proportional Reasoning

* Ratios:
* Regardless of how the objectives are stated in your curriculum concerning the ability to solve proportions or percent problems, the ultimate goal for your students should be focused on the development of proportional reasoning, not a collection of skills.
* It is useful to have a good idea of what constitutes a ratio and a proportion and in what contexts these mathematical ideas appear.
* Investigating contexts such as interests, taxes, and tips as well as connecting to work with similar figures, graphing, and slope.
* Types of Ratios:
* A ratio can be applied to a situation where the relative amounts of the quantities or measures are the same.
* To the student just beginning to develop an understanding of ratio, different settings or contexts may well seem like different ideas even though they are essentially the same from a mathematical viewpoint.
* Chinese teachers spend more time making sense of the subtle differences among fractions, ratios, and division, whereas US teachers connect ratios quickly to percents without discussion of these interrelated concepts.
* Table 18.1 in our book offers comparisons among fractions, ratios, and division similar to those used in Chinese lessons as prompts for students to discuss the relationships among these ideas.
* Part –to –whole ratios:
* Ratios can express comparisons of a part to whole for example:
* The ratio of the number of girls in a class to the number of students in the class.
* Part-to –Part Ratios:
* A ratio can also relate one part of a whole to another part of the same whole, for example:
* The number of girls in the class can be compared to the number of boys in the class.
* Rates as Ratios:
* Both part to whole and part to part ratios compare two measures of the same type of thing.
* A ratio can also be a rate.
* A rate is a comparison of the measures of two different things or quantities, the measuring unit is different for each value; For example:
* If 4 similar boats carry 36 passengers, then the comparison of 4 boats to 36 passengers is a ratio.
* Boats and passengers are different types of things.
* The ratio of passengers to boats is 36:4 which can also be written as 36/4 or 36 to 4.
* Other examples of rates as ratios are:
* Miles per gallon, square yards of wall coverage per gallon of paint, passengers per busload, and roses per bouquet are all rates.
* Proportional Reasoning
* It is both a qualitative and quantitative process.
* According to Lamon, the following are a few of the characteristics of proportional thinkers:
* Proportional thinkers have a sense of covariation. That is, they understand relationships in which two quantities vary together and are able to see how the variation in one coincides with the variation in another
* Proportional thinkers develop a wide variety of strategies for solving proportions or comparing ratios, most of which are based on informal strategies rather than prescribed algorithms.
* Proportional thinkers understand ratios as distinct entities representing a relationship different from the quantities they compare.
* We do not simply acquire the habits and skills of proportional reasoning simply by getting older.
* Lamon’s research and the research of others indicate that instruction that focuses on reasoning can have an effect on a student’s ability to reason proportionally, which begins early with multiplicative reasoning.
* How to help children develop proportional though process:

1. Provide ratio and proportion tasks in a wide range of contexts, including situations involving measurements, prices, geometric and other visual contexts, and rates of all sorts.
2. Encourage discussion and experimentation in predicting and comparing ratios. Help children distinguish between proportional and nonproportional comparisons by providing examples of each and discussing the differences.
3. Help children relate proportional reasoning to existing processes. The concept of unit fractions is very similar to unit rates. Research indicates that the use of a unit rate for comparing ratios and solving proportions is the most common approach among middle school students even when cross-product methods have been taught.
4. Recognize that symbolic or mechanical methods, such as the cross-product algorithm, for solving proportions do not develop proportional reasoning and should not be introduced until students have had many experiences with intuitive and conceptual methods.

* Identifying Multiplicative Relationships
* Students may confuse additive situations for multiplicative situations. Making explicit the type of relationships that exist between two values can greatly support students’ understanding of ratios and proportions.
* Consider the following sample problem
* Miller Middle School has 16 sixth-grade students, and 12 of them say that they are basketball fans. The remaining students are not basketball fans.
* Students are asked to describe whatever relationships they can between students who are basketball fans and who are not.
* Equivalent Ratios
* The focus should be on an intuitive rationale for why the pairs selected are in the same ratio.
* It is extremely important in these activities to include pairs of ratios that are not proportional but have a common difference.
* For example:
* 5/8 and 9/12 are not equivalent ratios, but the corresponding differences are the same: 8-5=12-9.
* Using contexts in comparing ratios helps students articulate their multiplicative or proportional thinking.
* Ratio Tables:
* Ratio tables or charts that show how two variable quantities are related are often good ways to organize information.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Acres | 5 | 10 | 15 | 20 | 25 |
| Pine trees | 75 | 150 | 225 |  |  |

* If the task is to find the number of trees for 65 acres of land or the number of acres needed to 750 trees, students can proceed by using addition. That is, they can add 5’s along the top row until they reach 65.
* This is a recursive pattern, or repeated addition.
* Algebra
* Graphing ratios provides a powerful connection to algebra.
* Proportional situations are linear situations, in fact, ratios are a special case of linear situations that will always go through the origin, since they are multiplicative relationships. The ratio or rate is the slope of the graph.
* Graphs provide another way of thinking about proportions, and they connect proportional thought to algebraic interpretations.
* Scale Drawings
* The connection between proportional reasoning and the geometric concept of similarity is very important. Similar figures provide a visual representation of proportions, and proportional thinking enhances the understanding of similarity.
* Discussion of the similar figures should focus on the ratios between and within the figures.
* Ideas for teaching scale drawings:
* If you wanted to make a scale model of the solar system and use a Ping-Pong ball for the earth, how far away should the sun be? How large a ball would you need?
* What scale should be used to draw a scale map of your city so that it will nicely fit onto a standard piece of poster board?
* Use the scale on a map to estimate the distance and travel time between two points of interest.